e have conservation of mass

$$+ \frac{\partial \rho}{\partial t} = 0,$$

discussed the total momentum also

$$= 0, \tag{3}$$

slit width.
ish so on the left side of (1) and (2)
; (1) and (2) we get

$$\eta_n \nabla \times (\nabla \times \mathbf{v}_n) + (2\eta_n + \eta')\nabla(\nabla \cdot \mathbf{v}_n). \tag{4}$$

arried by the normal fluid such that

$$\mathbf{v}_n \beta^{-1} \tag{5}$$

to be conserved, $\nabla \cdot \mathbf{q} = 0$ and

$$\nabla\beta = \mathbf{q} \cdot \nabla\beta. \tag{6}$$

$$(\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}$$

simplified to give

$$- \rho_s(\mathbf{v}_s \cdot \nabla)\mathbf{v}_s - \rho_n(\mathbf{v}_n \cdot \nabla)\mathbf{v}_n . \tag{7}$$

me assumptions which will later be
ke the $z$ axis along the length of the
ors in these directions are $\mathbf{e}_z$ and $\mathbf{e}_x$ .

$$T = T(z) \tag{8}$$

$\eta_n(z)$. We further assume that the
small compared to the other terms.
ted into $x$ and $z$ components to give[2]

$$\eta_n + \eta')\mathbf{q} \frac{d^2\beta}{dz^2} \tag{9}$$

he heat current density appears are not
we continue to use the boldface notation



and

$$\frac{\partial P}{\partial x} = (\eta_n + \eta') \frac{d\mathbf{q}}{dx} \frac{d\beta}{dz} . \tag{10}$$

Equation (9) may be solved if we assume that the second term on the right is small, i.e.

$$\beta^{-1} \frac{d^2\beta}{dz^2} \ll \frac{\eta_n}{2\eta_n + \eta'} \mathbf{q}^{-1} \frac{d^2\mathbf{q}}{dx^2} . \tag{11}$$

(Justification for this assumption will be given later.) Subject to the condition $\mathbf{q} = 0$ at the slit boundaries $\pm d/2$, the solution for $\mathbf{q}$ is

$$\mathbf{q} = \frac{3}{2} \bar{\mathbf{q}} \left(1 - \frac{4x^2}{d^2}\right). \tag{12}$$

Then the pressure gradient becomes

$$\frac{\partial P}{\partial z} = -\frac{12\eta_n \bar{\mathbf{q}}}{\rho s T d^2} . \tag{13}$$

This last equation is the basis of the so-called Allen-Reekie rule, which specifies that in the limit of small $\Delta T$'s the fountain pressure $P_f$ and the heat current density are proportional and that this relationship is independent of the form of $\mathbf{F}_{sn}$ . Since the right hand side of (13) is strongly temperature dependent, for larger temperature differences this equation must be integrated to give

$$\Delta P_z = P_f = -\int_{T_0}^{T_1} \frac{12\eta_n \bar{\mathbf{q}}}{\rho s T d^2} \frac{dz}{dT} dT. \tag{14}$$

In order to obtain the relationship between $P_f$ and $\bar{\mathbf{q}}$ for large temperature differences it is therefore necessary to obtain an expression for $dT/dz$ as a function of the temperature along the length of the slit. Since the temperature gradient along the slit does depend upon $\mathbf{F}_{sn}$ , as will be seen below, it is obvious that the relationship between $P_f$ and $\bar{\mathbf{q}}$ must for large temperature differences also depend upon $\mathbf{F}_{sn}$ .

We now wish to find an expression for the temperature gradient. To do so we must postulate a particular form for the frictional force $\mathbf{F}_{sn}$ . We shall concentrate our attention upon the Gorter-Mellink type of force, which we shall write in the slightly generalized form

$$\begin{aligned} \mathbf{F}_{sn}(\mathbf{v}_s - \mathbf{v}_n) &= A\rho_s\rho_n(|\mathbf{v}_s - \mathbf{v}_n| - \mathbf{v}_c)^{m-1}(\mathbf{v}_s - \mathbf{v}_n) \qquad |\mathbf{v}_s - \mathbf{v}_n| > \mathbf{v}_c \\ &= 0 \qquad |\mathbf{v}_s - \mathbf{v}_n| < \mathbf{v}_c . \end{aligned} \tag{15}$$

Here $A$ is the (temperature dependent) Gorter-Mellink coefficient, $\mathbf{v}_c$ is a (possibly temperature dependent) critical velocity, and $m$ has in various ex-